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Chiral Perturbation Theory with Photons and Leptons* †

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Abstract

I discuss a low-energy effective field theory which permits the full treatment of isospin-breaking effects in semileptonic weak interactions. In addition to the pseudoscalars and the photon, also the light leptons have to be included as dynamical degrees of freedom in an appropriate chiral Lagrangian. I describe the construction of the local action at next-to-leading order.

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1. In this talk, I would like to report about the status [1] of a research project on isospin-violating effects in the semileptonic decays of pions and kaons which is presently carried out by Marc Knecht, Heinz Rupertsberger, Pere Talavera and myself.

While isospin-breaking generated by a non-vanishing quark mass difference $m_d - m_u$ is fully contained in the pure QCD sector of the effective chiral Lagrangian [2], the analysis of isospin-violation of electromagnetic origin requires an extension of the usual low-energy effective theory. For purely pseudoscalar processes, the suitable theoretical framework for the three-flavour case has been worked out in [3, 4, 5] by including virtual photons and the appropriate local terms up to $\mathcal{O}(e^2p^2)$.

The treatment of electromagnetic corrections in semileptonic decays demands still a further extension of chiral perturbation theory. In this case, also the light leptons have to be included as explicit dynamical degrees of freedom. Only within such a framework, one will have full control over all possible isospin breaking effects in the analysis of new high statistics $K_{\ell 4}$ experiments by the E865 and KLOE collaborations at BNL [6] and DAΦNE [7], respectively. The same refined methods are, of course, also necessary for the interpretation of forthcoming high precision experiments on other semileptonic decays like $K_{\ell 3}$, etc.

2. To lowest order in the chiral expansion, the effective Lagrangian without dynamical photons and leptons (pure QCD) is nothing else than the non-linear sigma model in the presence of external vector, axial-vector, scalar and pseudoscalar sources v_μ, a_μ, $\chi = s+ip$. Following the notation of [8], it takes the form

$$\mathcal{L}_{\text{eff}} = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle, \tag{1}$$

where

$$\begin{aligned} u_\mu &= i[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger], \\ l_\mu &= v_\mu - a_\mu, \\ r_\mu &= v_\mu + a_\mu, \\ \chi_+ &= u^\dagger \chi u^\dagger + u\chi^\dagger u. \end{aligned} \tag{2}$$

Even this Lagrangian allows the treatment of electromagnetic or semileptonic processes as long as the photon or the leptons are occurring only as external fields. One simply takes external sources with the quantum numbers of the photon or the $W^\pm$.

For the description of dynamical photons and leptons, the extension of the lowest order Lagrangian (1) is rather easy. First of all, the photon field A_μ and the light leptons ℓ, ν_ℓ ($\ell = e, \mu$) are introduced in u_μ by adding appropriate terms to the external vector and axial-vector sources:

$$\begin{aligned} l_\mu &= v_\mu - a_\mu - eQ_L^{\text{em}} A_\mu + \sum_\ell (\bar{\ell}\gamma_\mu \nu_{\ell L} Q_L^{\text{w}} + \overline{\nu_{\ell L}} \gamma_\mu \ell Q_L^{\text{w}\dagger}), \\ r_\mu &= v_\mu + a_\mu - eQ_R^{\text{em}} A_\mu. \end{aligned} \tag{3}$$

The 3×3 matrices $Q_{L,R}^{\rm em}$, $Q_L^{\rm w}$ are additional spurion fields. At the end, one identifies $Q_{L,R}^{\rm em}$ with the quark charge matrix

$$Q^{\rm em} = \left[\begin{array}{ccc} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{array}\right], \tag{4}$$

whereas the weak spurion is taken at

$$Q_L^{\rm w} = -2\sqrt{2}\, G_F \left[\begin{array}{ccc} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right], \tag{5}$$

where G_F is the Fermi coupling constant and V_{ud}, V_{us} are Kobayashi–Maskawa matrix elements.

Then we have to introduce kinetic terms for the photon and the leptons and also an electromagnetic term of $\mathcal{O}(e^2p^0)$. With these building blocks, our lowest order effective Lagrangian takes the form

$$\begin{aligned} \mathcal{L}_{\rm eff} &= \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle + e^2F^4Z\langle \mathcal{Q}_L^{\rm em}\mathcal{Q}_R^{\rm em}\rangle \\ &\quad -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_\ell[\bar\ell(i\,\partial\!\!\!/ + eA\!\!\!/ - m_\ell)\ell + \overline{\nu_{\ell L}}\, i\,\partial\!\!\!/\nu_{\ell L}], \end{aligned} \tag{6}$$

where

$$\mathcal{Q}_L^{\rm em,w} := uQ_L^{\rm em,w}u^\dagger, \qquad \mathcal{Q}_R^{\rm em} := u^\dagger Q_R^{\rm em}u. \tag{7}$$

Finally, we have to define an extended chiral expansion scheme. The electric charge e, the lepton masses m_e, m_μ and fermion bilinears are considered (formally) as quantities of order p in the chiral counting, where p is a typical meson momentum. Note, however, that terms of $\mathcal{O}(e^4)$ will be neglected throughout.

3. As we are dealing with a so-called non-renormalizable theory, new local terms are arising at the next-to-leading-order. The associated coupling constants absorb the divergences of the one-loop graphs. Their finite parts are in principle certain functions of the parameters of the standard model. Because of our limited ability in solving the standard model (confinement problem), these low-energy constants have to be regarded as free parameters of our effective theory for the time being.

The list of local counterterms of our extended theory comprises, of course, the well-known Gasser–Leutwyler Lagrangian of $\mathcal{O}(p^4)$ [2] and the Urech Lagrangian of $\mathcal{O}(e^2p^2)$ [3] with the generalized l_μ and r_μ defined in Eq. (3). In the presence of virtual leptons, we have to introduce an additional "leptonic" Lagrangian [1]

$$\begin{aligned} \mathcal{L}_{\rm lept} &= e^2\sum_\ell\Big\{F^2\Big[X_1\bar\ell\gamma_\mu\nu_{\ell L}\langle u^\mu\{\mathcal{Q}_R^{\rm em},\mathcal{Q}_L^{\rm w}\}\rangle \\ &\qquad +X_2\bar\ell\gamma_\mu\nu_{\ell L}\langle u^\mu[\mathcal{Q}_R^{\rm em},\mathcal{Q}_L^{\rm w}]\rangle \end{aligned}$$

$$
\begin{aligned}
&\quad +X_3 m_\ell \overline{\ell}\nu_{\ell L}\langle \mathcal{Q}_L^{\rm w}\mathcal{Q}_R^{\rm em}\rangle \\
&\quad +iX_4 \overline{\ell}\gamma_\mu\nu_{\ell L}\langle \mathcal{Q}_L^{\rm w}\widehat{\nabla}^\mu\mathcal{Q}_L^{\rm em}\rangle \\
&\quad +iX_5 \overline{\ell}\gamma_\mu\nu_{\ell L}\langle \mathcal{Q}_L^{\rm w}\widehat{\nabla}^\mu\mathcal{Q}_R^{\rm em}\rangle + h.c.\Big] \\
&+X_6\overline{\ell}(i\not{\partial}+e\not{A})\ell \\
&+X_7 m_\ell\overline{\ell}\ell\Big\}\,. \qquad (8)
\end{aligned}
$$

where

$$
\begin{aligned}
\widehat{\nabla}_\mu\mathcal{Q}_L^{\rm em} &= \nabla_\mu\mathcal{Q}_L^{\rm em}+\frac{i}{2}[u_\mu,\mathcal{Q}_L^{\rm em}] = u(D_\mu Q_L^{\rm em})u^\dagger, \\
\widehat{\nabla}_\mu\mathcal{Q}_R^{\rm em} &= \nabla_\mu\mathcal{Q}_R^{\rm em}-\frac{i}{2}[u_\mu,\mathcal{Q}_R^{\rm em}] = u^\dagger(D_\mu Q_R^{\rm em})u, \qquad (9)
\end{aligned}
$$

with

$$
\begin{aligned}
D_\mu Q_L^{\rm em} &= \partial_\mu Q_L^{\rm em} - i[l_\mu, Q_L^{\rm em}], \\
D_\mu Q_R^{\rm em} &= \partial_\mu Q_R^{\rm em} - i[r_\mu, Q_R^{\rm em}]. \qquad (10)
\end{aligned}
$$

In $\mathcal{L}_{\rm lept}$ we consider only terms quadratic in the lepton fields and at most linear in G_F. The terms with $X_{4,5}$ will not appear in realistic physical processes as the generated amplitudes contain an external (axial-) vector source (see Eqs. (9) and (10)).

In deriving a minimal set of terms in Eq. (8), we have used partial integration, the equations of motion derived from the tree-level Lagrangian (6) and the relations

$$
\mathcal{Q}_L^{\rm em}\mathcal{Q}_L^{\rm w} = \frac{2}{3}\mathcal{Q}_L^{\rm w}, \qquad \mathcal{Q}_L^{\rm w}\mathcal{Q}_L^{\rm em} = -\frac{1}{3}\mathcal{Q}_L^{\rm w}, \qquad \langle \mathcal{Q}_L^{\rm w}\rangle = 0. \qquad (11)
$$

Finally, also a photon Lagrangian

$$
\mathcal{L}_\gamma = e^2 X_8 F_{\mu\nu}F^{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad (12)
$$

has to be added. This term cancels the divergences of the photon two-point function generated by the lepton loops.

The “new” low–energy couplings X_i arising here are divergent (except X_1). In the dimensional regularization scheme, they absorb the divergences of the one–loop graphs with internal lepton lines via the renormalization

$$
\begin{aligned}
X_i &= X_i^r(\mu) + \Xi_i\Lambda(\mu), \quad i = 1,\ldots,8, \\
\Lambda(\mu) &= \frac{\mu^{d-4}}{(4\pi)^2}\left\{\frac{1}{d-4} - \frac{1}{2}[\ln(4\pi)+\Gamma'(1)+1]\right\}. \qquad (13)
\end{aligned}
$$

The coefficients $\Xi_1,\ldots\Xi_7$ can be determined [1] by using super-heat-kernel methods [9, 10]:

$$
\begin{aligned}
&\Xi_1 = 0, \quad \Xi_2 = -\frac{3}{4}, \quad \Xi_3 = -3, \quad \Xi_4 = -\frac{3}{2}, \\
&\Xi_5 = \frac{3}{2}, \quad \Xi_6 = -5, \quad \Xi_7 = -1, \quad \Xi_8 = -\frac{4}{3}. \qquad (14)
\end{aligned}
$$

4. We have developed the appropriate low-energy effective theory for a complete treatment of isospin violating effects in semileptonic weak processes. The electromagnetic interaction requires the inclusion of the photon field and the light leptons as explicit dynamical degrees of freedom in the chiral Lagrangian. At next-to-leading order, the list of local terms given by Gasser and Leutwyler [2] for the QCD part and by Urech [3] for the electromagnetic interaction of the pseudoscalars has to be enlarged. This is, of course, a consequence of the presence of virtual leptons in our extended theory. Regarding pure lepton or photon bilinears as "trivial", five additional "non-trivial" terms of this type are arising. But two of them will not appear in realistic physical processes. One may therefore conclude that the main bulk of electromagnetic low-energy constants is already contained in Urech's Lagrangian and the inclusion of virtual leptons in chiral perturbation theory does not substantially aggravate the problem of unknown parameters.

As an illustration of the use of our effective theory, we have calculated [1] the decay rates of $\pi \to \ell\nu_\ell$ and $K \to \ell\nu_\ell$ including the electromagnetic contributions of $\mathcal{O}(e^2p^2)$. An investigation of the $K_{\ell 3}$ decays is presently in progress.

The continuation of our work will follow two principal lines. Firstly, we are now in the position to calculate the electromagnetic contributions to $K_{\ell 3}$ and $K_{\ell 4}$ decays where all constraints imposed by chiral symmetry are taken into account. In spite of our large ignorance of the actual values of the electromagnetic low-energy couplings, it will often be possible to relate the electromagnetic contributions to different processes. For specific combinations of observables one might even find parameter-free predictions. Simple examples of this kind have been given for the $P_{\ell 2}$ decays [1]. In some fortunate cases simple order-of-magnitude estimates for the electromagnetic couplings based on chiral dimensional analysis may even be sufficient.

Secondly, a further major task for the next future is, of course, the determination of the physical values of the electromagnetic low-energy coupling constants in the standard model. In contrast to the QCD low-energy couplings $L_1^r, \cdots L_{10}^r$ which are rather well determined from experimental input and large N_c arguments within the standard framework of chiral perturbation theory [2], only very little is known so far in the electromagnetic sector. First attempts to estimate some of the Urech constants K_i can be found in [11, 12, 13]. As far as the constants X_i are concerned, the recent analysis [14] of the counterterms contributing to the decay processes of light neutral pseudoscalars into charged lepton pairs raises hopes that reliable estimates for these constants can be achieved within a large-N_c approach.

References

[1] M. Knecht, H. Neufeld, H. Rupertsberger and P. Talavera, Chiral Perturbation Theory with Virtual Photons and Leptons, hep-ph/9909284, to appear in Eur. Phys. J. C.

[2] J. Gasser and H. Leutwyler, Chiral Perturbation Theory: Expansion in the Mass of the Strange Quark, Nucl. Phys. B **250**, 465 (1985).

[3] A. Urech, Virtual Photons in Chiral Perturbation Theory, Nucl. Phys. B **433**, 234 (1995).

[4] H. Neufeld and H. Rupertsberger, Isospin Breaking in Chiral Perturbation Theory and the Decays $\eta \to \pi\ell\nu$ and $\tau \to \eta\pi\nu$, Z. Phys. C **68**, 91 (1995).

[5] H. Neufeld and H. Rupertsberger, The Electromagnetic Interaction in Chiral Perturbation Theory, Z. Phys. C **71**, 131 (1996).

[6] M. Zeller, Kaon Decays, plenary talk at this meeting.

[7] G. Pancheri, Meson Physics at DAPHNE, plenary talk at this meeting.

[8] G. Ecker, J. Gasser, A. Pich and E. de Rafael, The Role of Resonances in Chiral Perturbation Theory, Nucl Phys. B **321**, 311 (1989).

[9] H. Neufeld, J. Gasser and G. Ecker, The One-Loop Functional as a Berezinian, Phys. Lett. B **438**, 106 (1998).

[10] H. Neufeld, The Super-Heat-Kernel Expansion and the Renormalization of the Pion–Nucleon Interaction, Eur. Phys. J. C **7**, 355 (1999).

[11] R. Baur and R. Urech, On the Corrections to Dashen's Theorem, Phys. Rev. D **53**, 6552 (1996).

[12] R. Baur and R. Urech, Resonance Contributions to the Electromagnetic Low-Energy Constants of Chiral Perturbation Theory, Nucl. Phys. B **499**, 319 (1997).

[13] B. Moussallam, A Sum Rule Approach to the Violation of Dashen's Theorem, Nucl. Phys. B **504**, 381 (1997).

[14] M. Knecht, S. Peris, M. Perrottet and E. de Rafael, Decay of Pseudoscalars into Lepton Pairs and Large-N_c QCD, hep-ph/9908283, to appear in Phys. Rev. Lett.